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Neutrino mixing and CP phase correlations

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ABSTRACT

A special form of the 3×3 Majorana neutrino mass matrix derivable from μ–τ interchange symmetry accompanied by a generalized *CP* transformation was obtained many years ago. It predicts $\theta_{23} = \pi/4$ as well as $\delta_{CP} = \pm\pi/2$, with $\theta_{13} \neq 0$. Whereas this is consistent with present data, we explore a deviation of this result which occurs naturally in a recent proposed model of radiative inverse seesaw neutrino mass.

A special form of the 3×3 Majorana neutrino mass matrix first appeared in 2002 [1,2], i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \tag{1}$$

where A, B are real. It was shown that $\theta_{13} \neq 0$ and yet both θ_{23} and the *CP* nonconserving phase δ_{CP} are maximal, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$. Subsequently, this pattern was shown [3] to be protected by a symmetry, i.e. $e \to e$ and $\mu \leftrightarrow \tau$ exchange with *CP* conjugation. All three predictions are consistent with present experimental data. Recently, a radiative (scotogenic) model of inverse seesaw neutrino mass has been proposed [4] which naturally obtains

$$\mathcal{M}_\nu^\lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix} \mathcal{M}_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix}, \tag{2}$$

where $\lambda = f_\tau / f_\mu$ is the ratio of two real Yukawa couplings.

This model has three real singlet scalars $s_{1,2,3}$ and one Dirac fermion doublet (E^0, E^-) and one Dirac fermion singlet N, all of which are odd under an exactly conserved (dark) Z_2 symmetry. As a result, the third one-loop radiative mechanism proposed in 1998 [5] for generating neutrino mass is realized, as shown in Fig. 1.

The mass matrix linking $(\bar{N}_L, \bar{E}_L^0)$ to (N_R, E_R^0) is given by

$$\mathcal{M}_{N,E} = \begin{pmatrix} m_N & m_D \\ m_F & m_E \end{pmatrix}, \tag{3}$$

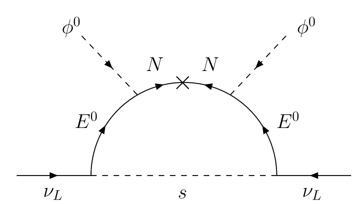

Fig. 1. One-loop generation of inverse seesaw neutrino mass.

where m_N, m_E are invariant mass terms, and m_D, m_F come from the Higgs vacuum expectation value $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, N and E^0 mix to form two Dirac fermions of masses $m_{1,2}$, with mixing angles

$$m_D m_E + m_F m_N = \sin\theta_L \cos\theta_L (m_1^2 - m_2^2), \tag{4}$$

$$m_D m_N + m_F m_E = \sin\theta_R \cos\theta_R (m_1^2 - m_2^2). \tag{5}$$

To connect the loop, Majorana mass terms $(m_L/2) N_L N_L$ and $(m_R/2) N_R N_R$ are assumed. Since both E and N may be defined to carry lepton number, these new terms violate lepton number softly and may be naturally small, thus realizing the mechanism of inverse seesaw [6–8] as explained in Ref. [4]. Using the Yukawa interaction $f s \bar{E}_R^0 \nu_L$, the one-loop Majorana neutrino mass is given by

$$\begin{aligned} m_\nu = {} & f^2 m_R \sin^2\theta_R \cos^2\theta_R (m_1^2 - m_2^2)^2 \\ & \times \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2} \frac{1}{(k^2 - m_2^2)^2} \\ & + f^2 m_L m_1^2 \sin^2\theta_L \cos^2\theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2} \end{aligned}$$

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$$+ f^2 m_L m_2^2 \sin^2\theta_R \cos^2\theta_L \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_2^2)^2}$$
$$- 2 f^2 m_L m_1 m_2 \sin\theta_L \sin\theta_R \cos\theta_L \cos\theta_R$$
$$\times \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)} \frac{1}{(k^2 - m_2^2)}. \quad (6)$$

It was also shown in Ref. [4] that the implementation of a discrete flavor Z_3 symmetry, which is softly broken by the 3×3 real scalar mass matrix spanning $s_{1,2,3}$, leads to $\mathcal{M}_\nu^\lambda$ of Eq. (2).

To explore how the predictions $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$ are changed for $\lambda \neq 1$, consider the general diagonalization of $\mathcal{M}_\nu$, i.e.

$$\mathcal{M}_\nu = E_\alpha U E_\beta \mathcal{M}_d E_\beta U^T E_\alpha, \quad (7)$$

where

$$E_\alpha = \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix}, \qquad E_\beta = \begin{pmatrix} e^{i\beta_1} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & e^{i\beta_3} \end{pmatrix},$$
$$\mathcal{M}_d = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \quad (8)$$

Hence

$$\mathcal{M}_\nu \mathcal{M}_\nu^\dagger = E_\alpha U \mathcal{M}_d^2 U^\dagger E_\alpha^\dagger. \quad (9)$$

We then have

$$\mathcal{M}_\nu^\lambda (\mathcal{M}_\nu^\lambda)^\dagger = E_\alpha U [1 + \Delta] \mathcal{M}_{\lambda d}^2 [1 + \Delta^\dagger] U^\dagger E_\alpha^\dagger, \quad (10)$$

where

$$\Delta = U^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda - 1 \end{pmatrix} U, \qquad \mathcal{M}_{\lambda d}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & \lambda^2 m_3^2 \end{pmatrix}. \quad (11)$$

We now diagonalize numerically

$$[1 + \Delta] \mathcal{M}_{\lambda d}^2 [1 + \Delta^\dagger] = O \mathcal{M}_{new}^2 O^T, \quad (12)$$

where O is an orthogonal matrix, and $\mathcal{M}_{new}^2$ is diagonal with mass eigenvalues equal to the squares of the physical neutrino masses. Let us define

$$A = (1 + \Delta)^{-1} O, \quad (13)$$

then

$$A \mathcal{M}_{new}^2 A^\dagger = \mathcal{M}_{\lambda d}^2. \quad (14)$$

Since U is known with $\theta_{23} = \pi/4$ and $\delta = \pm\pi/2$, we know Δ once λ is chosen. The orthogonal matrix O has three angles as parameters, so A has three parameters. In Eq. (14), once the three physical neutrino mass eigenvalues of $\mathcal{M}_{new}^2$ are given, the three off-diagonal entries of $\mathcal{M}_{\lambda d}^2$ are constrained to be zero, thus determining the three unknown parameters of O. Once O is known, UO is the new neutrino mixing matrix, from which we can extract the correlation of θ_{23} with δ_{CP}. There is of course an ambiguity in choosing the three physical neutrino masses, since only Δm_{32}^2 and Δm_{21}^2 are known. There are also the two different choices of $m_1 < m_2 < m_3$ (normal ordering) and $m_3 < m_1 < m_2$ (inverted ordering). We consider each case, and choose a value of either m_1 or m_3 starting from zero. We then obtain numerically the values of $\sin^2(2\theta_{23})$ and δ_{CP} as functions of $\lambda \neq 1$. We need also to adjust the input values of θ_{12} and θ_{13}, so that their output values for $\lambda \neq 1$ are the preferred experimental values.

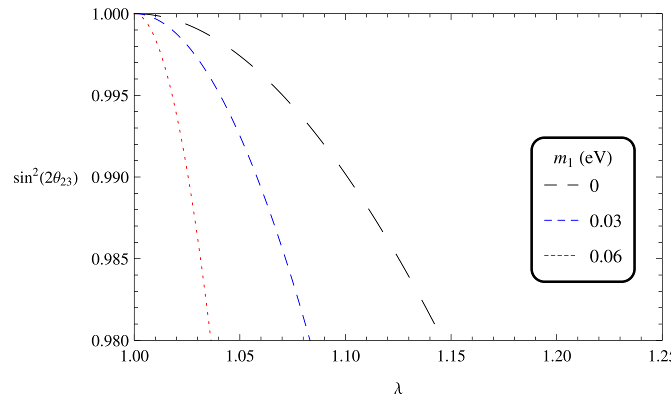

Fig. 2. $\sin^2(2\theta_{23})$ versus λ in normal ordering.

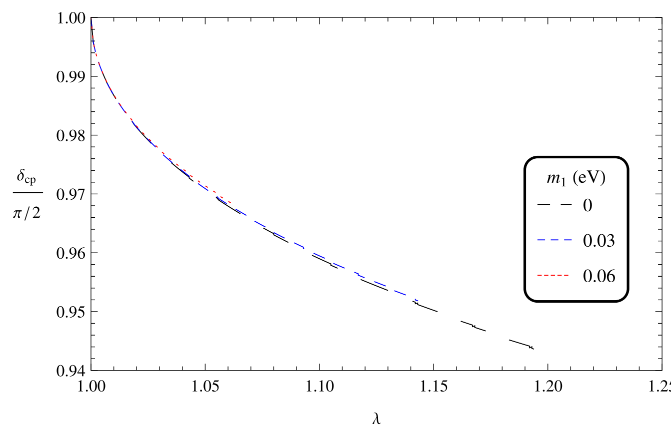

Fig. 3. δ_{CP} versus λ in normal ordering.

We use the 2014 Particle Data Group values [9] of neutrino parameters:

$$\sin^2(2\theta_{12}) = 0.846 \pm 0.021,$$
$$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \quad (15)$$
$$\sin^2(2\theta_{23}) = 0.999 \begin{pmatrix} +0.001 \\ -0.018 \end{pmatrix},$$
$$\Delta m_{32}^2 = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2 \text{ (normal)}, \quad (16)$$
$$\sin^2(2\theta_{23}) = 1.000 \begin{pmatrix} +0.000 \\ -0.017 \end{pmatrix},$$
$$\Delta m_{32}^2 = (2.52 \pm 0.07) \times 10^{-3} \text{ eV}^2 \text{ (inverted)}, \quad (17)$$
$$\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}. \quad (18)$$

We consider first normal ordering, choosing the three representative values $m_1 = 0, 0.03, 0.06$ eV. We then vary the value of $\lambda > 1$. [The case $\lambda < 1$ is equivalent to $\lambda^{-1} > 1$ with μ–τ exchange.] Following the algorithm already mentioned, we obtain numerically the values of $\sin^2(2\theta_{23})$ and δ_{CP} as functions of λ. Our solutions are fixed by the central values of Δm_{21}^2, Δm_{32}^2, $\sin^2(2\theta_{12})$, and $\sin^2(2\theta_{13})$. In Figs. 2 and 3 we plot $\sin^2(2\theta_{23})$ and δ_{CP} respectively versus λ. We see from Fig. 2 that $\lambda < 1.15$ is required for $\sin^2(2\theta_{23}) > 0.98$. We also see from Fig. 3 that δ_{CP} is not sensitive to m_1. Note that our scheme does not distinguish δ_{CP} from $-\delta_{CP}$. In Fig. 4 we plot $\sin^2(2\theta_{23})$ versus δ_{CP}. We see that $\delta_{CP}/(\pi/2) > 0.95$ is required for $\sin^2(2\theta_{23}) > 0.98$.

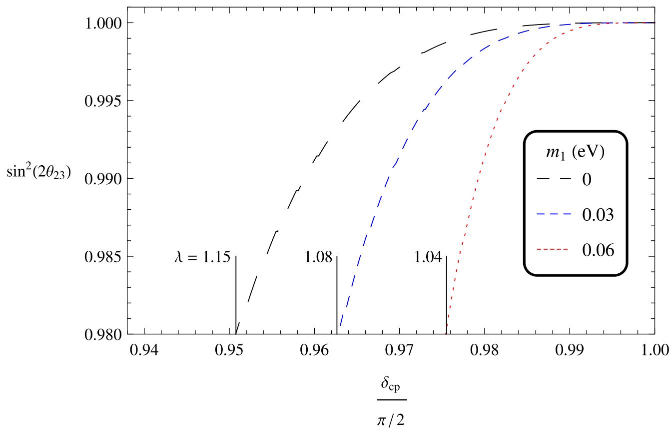

Fig. 4. $\sin^2(2\theta_{23})$ versus δ_{CP} in normal ordering.

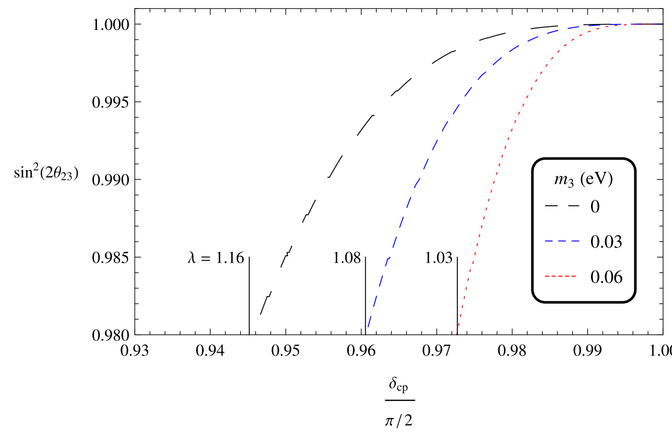

Fig. 7. $\sin^2(2\theta_{23})$ versus δ_{CP} in inverted ordering.

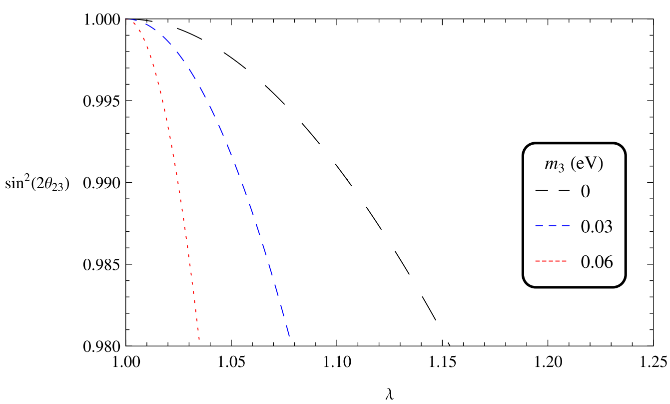

Fig. 5. $\sin^2(2\theta_{23})$ versus λ in inverted ordering.

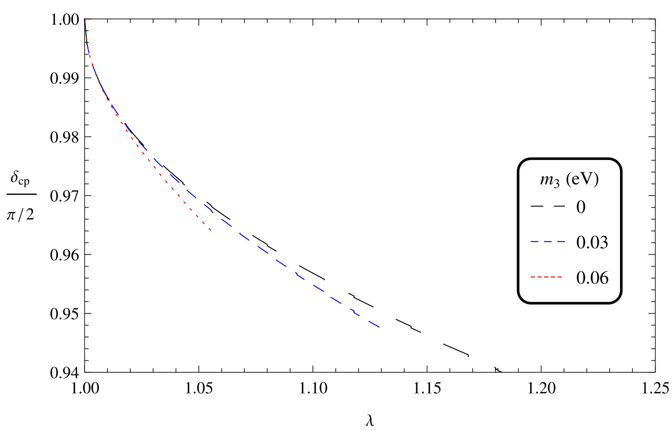

Fig. 6. δ_{CP} versus λ in inverted ordering.

We then consider inverted ordering, using m_3 instead of m_1. We plot in Figs. 5, 6, and 7 the corresponding results. Note that in our scheme, the effective neutrino mass m_{ee} measured in neutrinoless double beta decay is very close to m_1 in normal ordering and $m_3 + \sqrt{\Delta m^2_{32}}$ in inverted ordering. We see similar constraints on $\sin^2(2\theta_{23})$ and δ_{CP}. In other words, our scheme is insensitive to whether normal or inverted ordering is chosen. Finally, we have checked numerically that $\theta_{23} < \pi/4$ if $\lambda > 1$, and $\theta_{23} > \pi/4$ if $\lambda < 1$. As we already mentioned, the two solutions are related by the mapping $\lambda \to \lambda^{-1}$.

In conclusion, we have explored the possible deviation from the prediction of maximal θ_{23} and maximal δ_{CP} in a model of radiative inverse seesaw neutrino mass. We find that given the present 1σ bound of 0.98 on $\sin^2(2\theta_{23})$, $\delta_{CP}/(\pi/2)$ must be greater than about 0.95.

Acknowledgements

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